Station Functions and Their Application to Engineering Problems

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A description of the station function method, derivation of the functions, and application of the technique to polynomial solutions of nonlinear problems is given. Five examples of applications of the functions are presented showing, in different ways, how the boundary conditions can be defined in satisfying the requirements of the derived station functions. These examples probably encompass all general aperiodic functions likely to occur in engineering practice.

Nomenclature

$y_1, y_2, \ldots, y_n$ = ordinate or influence coefficient taken at reference stations noted by subscripts
$x_1, x_2, \ldots, x_n$ = reference stations
$a_{1i}, a_{2i}, \ldots, a_{(n+1)i}$ = influence coefficients in terms of station functions
$f(x)$ = station function
$f(y)$ = discrete function
i = ith station and specific function such that $f_i(i) = 1$
j = jth station of the function where $f_i(j) = 0$ and where $j \neq i$
$f_i'(n)$ = first derivative of ith function at nth station
$|[A]|$ = determinant of complementary function matrix
a_{ki} = station function coefficient of kth row, ith column
$[A^{-1}] = [A_k]'/[A]$ = reciprocal matrix of $[A]$
$[A_k]'$ = adjoint matrix of $[A]$
$[I] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ = unit matrix
v = helicopter rotor induced velocity, fps
V_t = rotor blade tip speed, fps
a_x = lift curve slope blade spanwise station, x
$\sigma = Bc/\pi R$ = rotor solidity factor
B = number of blades
c = blade chord
R = blade radius
θ_o = blade root collective pitch angle
θ_t = blade twist, negative for washout
$x = r/R$ = blade station at radius r
ρ = air density
T = thrust
ihp = induced horsepower
a,b,c,d,e,f = coefficients of Eqs. (9a) and (9b)
M = Mach number
M_{CR} = critical Mach number
c_d = section profile drag coefficient
$c_{d\,\mathrm{inc}}$ = incompressible section profile drag coefficient
c_n = section normal force coefficient
c_l = section lift coefficient
ϕ = inflow angle at blade section
α = angle of attack at blade section
θ = blade pitch angle at section
θ_2' = longitudinal stick position, in.
θ_2 = longitudinal blade pitch angle, deg

Subscripts

$1, 2, \ldots, n$ = reference stations

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Introduction

THE concept of station functions in analysis was first introduced by Rauscher,[1] who describes a practical approach in the determining of interpolation curves characteristic of those associated with flutter and vibration analyses. The purpose of this paper is to show that the station function technique is quite flexible and may be applied with considerable success to many other engineering problems as well.

In application, station functions may be used in determining distribution curves. They are unique in that the functions are applicable to families of such curves and are, in fact, independent of the magnitude of the influence coefficients affecting each distribution curve. The discrete function can be of such simple form that little complexity is involved in describing it as a distribution, or as a differential or integral of the function. One of the principles of application shown herein describes how pre-integration may be performed in conjunction with the station-function equations even before values of the particular influence coefficients are determined.

The prerequisites in establishing the station-function equations that will uniquely describe the discrete function are knowledge of the boundary conditions to be satisfied, and the stations at which the most important influence coefficients are likely to occur, i.e., stations at which occur the greatest influence upon the curvature of the distribution curve, a discontinuity in the distribution, or the like. Having once established the proper stations at which the station-function equations are to be applicable, the determined station functions are then descriptive of a family of such curves, all bearing somewhat similar characteristics. It is to be noted, however, that because of the interpolation characteristic of station functions, considerable freedom may be experienced in their use without any particular loss in accuracy. For example, one discrete function may have a reverse curvature in distribution which happens to be influenced at the particular stations chosen for the set of station functions, whereas another discrete function may be, say, divergent or somewhat exponential in character. It is entirely possible that the one set of station functions applies equally well to both types of distribution curves.

The importance in the use of station functions for determining interpolation curves lies in the fact that less work is involved. Having once established a set or sets of station functions, they may be used over and over again, independent of values of the influence coefficients. Two to four stations are all that are needed even for some of the most complicated of distribution curves. A discrete function is obtained by determining values of the influence coefficients only at the stations for which the station functions were previously established.

In applying station functions to helicopter performance calculations, for instance, it is shown subsequently that no more than three stations are required in determining the distribution curves from which thrust and induced power are obtained. The deviations in values performed by the station function method are less than 1.5% of those performed by the more exacting method that employs nine stations along the rotor blade. The three stations are necessary for blades having twist. Actually, for untwisted blades only two stations are required in determining the distribution curves. It is noteworthy that only four stations are required in obtaining an excellent interpolation curve of Fig. 1, which describes a typical drag rise curve from subcritical through supercritical Mach numbers. In practice, it has been found that once a discrete interpolation function has been obtained, the constants of the distribution equation may be rounded to slide rule accuracy, and subsequent calculations performed therefrom have been found to be satisfactory for most engineering purposes. However, in determining the discrete function it is advisable to use a desk calculator in order that a sufficient number of significant figures be available when the differences are taken.

Included in this paper are twelve sets of functions that have been found to be particularly useful. They were developed from time to time as the need arose and have been used in solutions of many engineering problems since 1953.

Mathematical Development

Basic Theory

The concept of station functions is the fitting of interpolation curves descriptive of the actual boundary conditions of the problem, including the requirement of proper tie-ins between the slopes, curvatures, and rates of change of curvature of the function on both sides of each reference station. A rigorous application of this concept usually results in a high-degree polynomial for use as an interpolation function of the problem. Certain approximations and eliminations of boundary conditions may be made, however, which reduce the degree of the interpolation function while still maintaining a high degree of accuracy considered within practicable applications.

In the fitting of an interpolation function $f(x)$ to the ordinates $y_1, y_2, \ldots, y_n$ at a series of reference stations $x_1, x_2, \ldots, x_n$, the function assumes the form

$$f(x) = y_1f_1(x) + y_2f_2(x) + \ldots + y_nf_n(x) \qquad (1)$$

with $f_1(x), f_2(x), \ldots, f_n(x)$ functions of x determined by the boundary conditions but independent of $y_1, y_2, \ldots, y_n$ such that

$$f_i(x) = a_{1i}x + a_{2i}x^2 + \ldots + a_{(n+1)i}x^{(n+1)} \qquad (2)$$

and thus each of the reference ordinates $y_1, y_2, \ldots, y_n$ makes its own contribution to $f(x)$. As the function $f(x)$ must satisfy the boundary conditions regardless of the values $y_1, y_2, \ldots, y_n$, the component function $f_i(x)$ must individually satisfy those conditions.

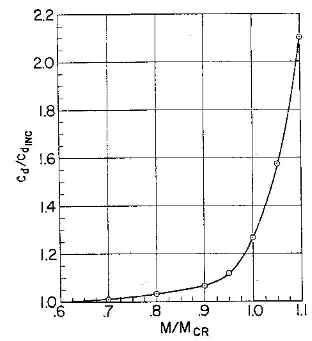

Fig. 1 Typical drag rise curve from subcritical through supercritical Mach numbers.

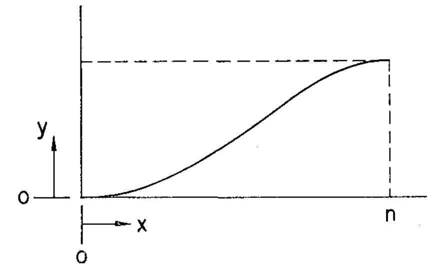

Fig. 2 Graphic illustration of station-function application.

In order that the function $f(x)$ may have the value y_1, at the station x_1, irrespective of the values $y_2, \ldots, y_n$, it is necessary that $f_1(x_1) = 1$ and $f_2(x_1) = \ldots = f_n(x_1) = 0$. Similarly, it is necessary that each of the other component functions has a value of unity at the station considered and values of zero at all the other stations. Each component function, then, may be considered as a kind of extension function for the station ordinate with which it is associated. The values of these continuous component functions at the reference stations must equal the ordinates or influence factors of the reference stations. The interpolation function is therefore a continuous function of the influence factors of the reference stations. The procedure results in n homogeneous equations in n influence factors of the reference stations. For example, consider Fig. 2 for which the following conditions may be said to exist:

$$\begin{array}{lll} \text{a)}\ x = 0 & f_i(0) & = 0 \\ \text{b)}\ x = n & f_i'(n) & = 0 \\ \text{c)}\ x = i & f_i(i) & = 1 \\ \text{d)}\ x = j & f_i(j) & = 0 \end{array}$$

where

$$j \neq i$$

where a and b are the boundary conditions, and c and d are the conditions imposed by the station functions. From Eq. (2), then, values of the coefficients a_{ij}, which satisfy the foregoing conditions, may be determined. Thus,

$$\begin{aligned} f_i'(n) &= 0 = a_{1i} + 2na_{2i} + \ldots + (n+1)n^na_{(n+1)i} \\ f_i(i) &= 1 = ia_{1i} + i^2a_{2i} + \ldots + i^{(n+1)}a_{(n+1)i} \\ f_i(j) &= 0 = ja_{1i} + j^2a_{2i} + \ldots + j^{(n+1)}a_{(n+1)i} \end{aligned} \qquad (3)$$

where

$$j \neq i$$

Theoretical Application

A brief illustration in application of the theory just mentioned is given here. For example, let it be assumed that functions for three stations are desired. This signifies that a third-degree polynomial representing the three functions will result. Further, it is convenient, in writing two equations in j, to select values of each in a symmetric manner from station to station. This does not mean that the stations must be equidistant but that their complementary functions be symmetrically composed.

In illustration of this theoretical application, the following conditions may be said to exist:

$$\begin{array}{ll} x = 0 & f_i(0) = 0 \\ x = i & f_i(i) = 1 \\ x = j & f_i(j) = 0 \\ & j \neq i \end{array}$$

Then from Eq. (2),

$$\begin{aligned} f_i(x) &= 0 = a_{1i}x + a_{2i}x^2 + a_{3i}x^3 \\ f_i(i) &= 1 = a_{1i}i + a_{2i}i^2 + a_{3i}i^3 \\ f_i(j) &= 0 = a_{1i}j + a_{2i}j^2 + a_{3i}j^3 \end{aligned} \qquad (4)$$

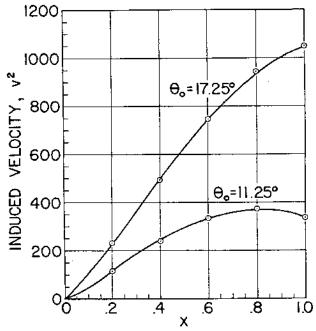

Fig. 3 Square of induced velocity vs span ratio for two collective pitch angles.

Since, for this example, the required number of stations is three, the following symmetric relations can be made:

$$\begin{array}{lll} i = x_1 & j_1 = x_2 & j_2 = x_3 \\ i = x_2 & j_1 = x_3 & j_2 = x_1 \\ i = x_3 & j_1 = x_1 & j_2 = x_2 \end{array}$$

then the complementary function $|[A]|$ of Eq. (4) is the same for all the reference stations. Thus,

$$|[A]|_1 = |[A]|_2 = |[A]|_3 = |[A]| = \begin{vmatrix} x_1 & x_1^2 & x_1^3 \\ x_2 & x_2^2 & x_2^3 \\ x_3 & x_3^2 & x_3^2 \end{vmatrix}$$

The particular solutions for the constants a_{ki} may be obtained from

$$[a_{ki}] = [A^{-1}][I] \qquad (5)$$

where

$$[a_{ki}] = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$[A^{-1}] = \frac{[A_k]'}{[A]}$$

$$[I] = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

Equation (5) is the solution of Eq. (4) which results in three third-degree polynomials, one for each function $f(x)$.

Numerical Application

Let

$$x_1 = 0.30 \qquad x_2 = 0.65 \qquad x_3 = 1.00$$

Then

$$\begin{array}{lll} i = 0.30 & j_1 = 0.65 & j_2 = 1.00 \\ i = 0.65 & j_1 = 1.00 & j_2 = 0.30 \\ i = 1.00 & j_1 = 0.30 & j_2 = 0.65 \end{array}$$

and the complementary function $[A]$ of Eq. (4) is

$$|[A]| = \begin{vmatrix} 0.3000 & 0.0900 & 0.0270 \\ 0.6500 & 0.4225 & 0.2746 \\ 1 & 1 & 1 \end{vmatrix} = 0.01672$$

From Eq. (5), then

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \frac{\begin{bmatrix} 0.1479 & -0.0630 & 0.01310 \\ -0.3754 & 0.2730 & -0.06483 \\ 0.2275 & -0.2100 & 0.06825 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}{0.01672}$$

since

$$\begin{aligned} f_1(x) &= a_{11}x + a_{21}x^2 + a_{31}x^3 \\ f_2(x) &= a_{12}x + a_{22}x^2 + a_{32}x^3 \\ f_3(x) &= a_{13}x + a_{23}x^2 + a_{33}x^3 \end{aligned}$$

Therefore

$$\begin{aligned} f_1(x) &= 8.846x - 22.45x^2 + 13.61x^3 \\ f_2(x) &= -3.768x + 16.33x^2 - 12.56x^3 \\ f_3(x) &= 0.7960x - 3.877x^2 + 4.082x^3 \end{aligned} \qquad (6)$$

Equation (6) gives the functions for the three stations in x; 0.30, 0.65, and 1.0. To arrive at a final function representative of the given curve, a product of the influence coefficient and representative function is made. All coefficients in like powers of x are then summed. The resulting equation, therefore, is the final function $f(y)$ desired.

It should be noted of Eq. (6), as a typical set of third-degree functions, that for $x = 0.30$, $f_1(x) = 1.0$, while $f_2(x) = f_3(x) = 0$, and for $x = 0.65$, $f_2(x) = 1.0$, while $f_1(x) = f_3(x) = 0$, and so forth.

Engineering Applications

Hovering Thrust and Power of Helicopter Rotor

An application of the station-function concept presented herein results in a more rapid method for obtaining hovering thrust and induced power of a helicopter rotor than the analytical procedures previously used and retains the same degree of accuracy as previously enjoyed.

The unusual number of induced velocity calculations involve nine stations along the blade radius. It is shown that only three stations are required through use of station functions to determine the induced velocity of a twisted blade. Actually, only two stations are necessary for a nontwisted blade.

The definition of the induced velocity employed here is that given by Stepniewski[2] where

$$v = V_t\left[-\frac{a_x\sigma}{16} + \left\{\left(\frac{a_x\sigma}{16}\right)^2 + \frac{a_x\sigma x}{8}(\theta_0 + \theta_t x)\right\}^{1/2}\right] \qquad (7)$$

The thrust and induced power are obtained by

$$\begin{aligned} T &= 4\rho\pi R^2 \int_{x_1}^{x_2} v^2 x dx \\ \text{ihp} &= \frac{4\rho\pi R^2}{550} \int_{x_1}^{x_2} v^3 x dx \end{aligned} \qquad (8)$$

By use of the station-function technique, pre-integration of Eq. (8) may be performed. Thus,

$$T = 4\rho\pi R^2[(0.294)a + (0.212)b + (0.163)c] \qquad (9a)$$

$$\text{ihp} = \frac{4\rho\pi R^2}{550}[(0.294)d + (0.212)e + (0.163)f] \qquad (9b)$$

where the integrations of Eq. (9a) and (9b) were performed

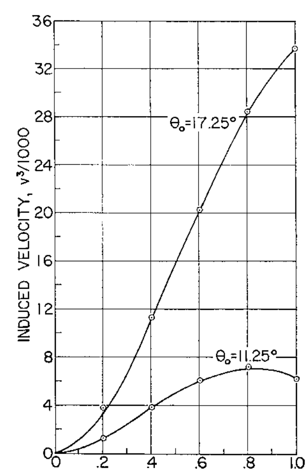

Fig. 4 Cube of induced velocity vs span ratio for two collective pitch angles.

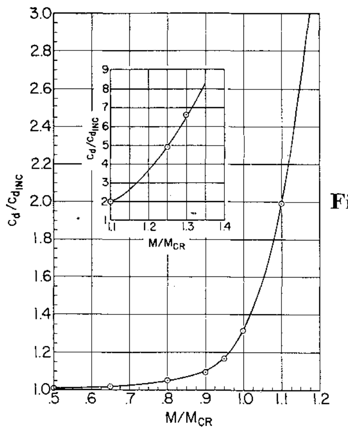

Fig. 5 Effect of compressibility on profile drag.

between the limits of $x_2 = 0.96$ and $x_1 = 0.15$. The coefficients of Eq. (9a) and (9b) are given as follows:

$$\left.\begin{aligned} a &= v_1^2a_1 + v_2^2a_2 + v_3^2a_3 \\ b &= v_1^2b_1 + v_2^2b_2 + v_3^2b_3 \\ c &= v_1^2c_1 + v_2^2c_2 + v_3^2c_3 \end{aligned}\right\} \qquad (10a)$$

$$\left.\begin{aligned} d &= v_1^3a_1 + v_2^3a_2 + v_3^3a_3 \\ e &= v_1^3b_1 + v_2^3b_2 + v_3^3b_3 \\ f &= v_1^3c_1 + v_2^3c_2 + v_3^3c_3 \end{aligned}\right\} \qquad (10b)$$

If the coefficients of the station functions of Eq. (6) are used, then

$$\begin{aligned} a_1 &= 8.846 & b_1 &= -22.45 & c_1 &= 13.61 \\ a_2 &= -3.768 & b_2 &= 16.33 & c_2 &= -12.56 \\ a_3 &= 0.7960 & b_3 &= -3.877 & c_3 &= 4.082 \end{aligned}$$

The induced velocities v_1, v_2, v_3, are induced velocities calculated by Eq. (7) at the three spanwise stations specified as follows:

$$\begin{aligned} v_1 &= \text{induced velocity } v \text{ at } x_1 = 0.30 \\ v_2 &= \text{induced velocity } v \text{ at } x_2 = 0.65 \\ v_3 &= \text{induced velocity } v \text{ at } x_3 = 1.0 \end{aligned}$$

By squaring and cubing these induced velocities and inserting them in Eq. (10a) and (10b), the hovering thrust and induced horsepower are obtained from Eq. (9a) and (9b).

Figures 3 and 4 illustrate the accuracies of the station-function method. The solid lines represent the previous method of analysis wherein nine stations along the blade radius were used. The circled points are those calculated by the station-function technique using but three stations specified by Eq. (6). The station-function method provides good agreement with the previous method with values of the thrust and induced horsepower within 1.5%.

Typical Drag Rise Curve from Subcritical through Supercritical Mach Numbers

Figure 1 illustrates a typical drag rise curve from the subcritical through the supercritical Mach numbers. The solid curve is the function, and the circled points are those

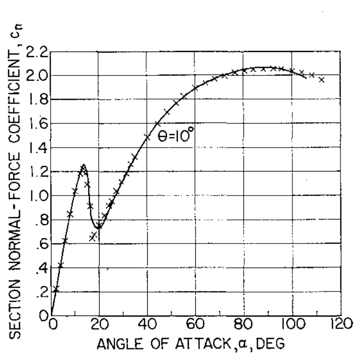

Fig. 6 Section normal-force coefficient of an NACA 0012 airfoil.

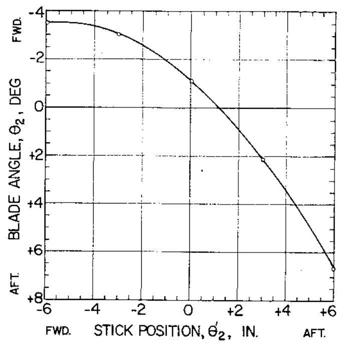

Fig. 7 A helicopter longitudinal-control variation.

values obtained from the station functions given in Table 9.

Since the boundary condition for the station functions is obviously $y = 0$ when $x = 0$, it is necessary that the abscissa and ordinate values of Fig. 1 be arranged to meet this boundary condition. This was easily accomplished by letting

$$\begin{aligned} x &= 2(M/M_{CR} - 0.6) \\ y &= (c_d/c_{d\text{inc}} - 1) \end{aligned}$$

Thus

$$\begin{aligned} c_d/c_{d\text{inc}} &= F(M/M_{CR}) \\ &= 1.0 + y_1f_1(x) + y_2f_2(x) + \ldots + y_4f_4(x) \end{aligned} \qquad (11)$$

In numerical form Eq. (11) is

$$\frac{c_d}{c_{d\text{inc}}} = \left[28.2060 - 147.542\left(\frac{M}{M_{CR}}\right) + 297.478\left(\frac{M}{M_{CR}}\right)^2 - 264.472\left(\frac{M}{M_{CR}}\right)^3 + 87.6000\left(\frac{M}{M_{CR}}\right)^4\right. \qquad (12)$$

which gives the values for the circled points in Fig. 1. Excellent agreement with the Mach number curve resulted.

Compressibility Correction on Profile Drag

Figure 5 illustrates another application of the station-function technique in curve fitting an algebraic polynomial to a characteristic effect on drag due to compressibility. This example was selected further to illustrate means for handling the boundary values. By use of the following functions:

$$\begin{aligned} f_1(x) &= 5.00135x - 6.41718x^2 + 2.95865x^3 - 0.583451x^4 + 0.0416722x^5 \\ f_2(x) &= -5.00083x + 8.91815x^2 - 4.91748x^3 + 1.08351x^4 - 0.0833472x^5 \\ f_3(x) &= 3.33389x - 6.50108x^2 + 4.08401x^3 - 1.00017x^4 + 0.0833472x^5 \\ f_4(x) &= -1.25021x + 2.54209x^2 - 1.70862x^3 + 0.458410x^4 - 0.0416736x^5 \\ f_5(x) &= 0.200033x - 0.416736x^2 + 0.291715x^3 - 0.0833472x^4 + 0.00833472x^5 \end{aligned} \qquad (13)$$

and by letting

$$x = \frac{20}{3}\left(\frac{M}{M_{CR}} - 0.5\right)$$

$$y = \left(\frac{c_d}{c_{d\text{inc}}} - 1.009\right)$$

the resulting polynomial in x and y becomes

$$y = [-0.098064x + 0.186828x^2 - 0.093562x^3 + 0.01270993x^4 + 0.001091838x^5] \qquad (14)$$

and the final equation becomes

$$\frac{c_d}{c_{d\text{inc}}} = \left[7.996816 - 37.808638\left(\frac{M}{M_{\text{CR}}}\right) + 69.572985\left(\frac{M}{M_{\text{CR}}}\right)^2 - 41.988859\left(\frac{M}{M_{\text{CR}}}\right)^3 - 10.839249\left(\frac{M}{M_{\text{CR}}}\right)^4 + 14.378113\left(\frac{M}{M_{\text{CR}}}\right)^5\right] \quad (15)$$

Equation (15) is almost an exact representation of the given curve showing the effect of compressibility on profile drag between the limits M/M_{CR} of 0.5–1.3 as illustrated by the circled points on Fig. 5.

Station Function Analysis of a Normal Force Curve

In the course of helicopter analyses, the determining of the rotor blade flapping motion due to large control inputs generally requires the use of the nonlinear inflow functions in order that the resulting calculated motion be similar to that recorded experimentally.

It has been found that the use of station functions for polynomial definition of the blade aerodynamic normal force coefficient has materially simplified and shortened this calculation procedure.

The blade of a helicopter in forward flight experiences a region of reversed flow. The aerodynamic sequence of events of such a phenomenon is similar to the results published by NACA.[3] Figure 6 shows the results of these tests in terms of the disk normal force coefficient referred to the inflow angle at the blade element. This normal force coefficient was obtained from

$$c_n = c_l \cos\phi + c_d \sin\phi \quad (16)$$

where $\phi = \alpha - \theta$. The solid line of Fig. 6 represents the variation of the normal force coefficient with angle of attack for collective pitch angle of 10°. The cross points illustrate the polynomial representation obtained by the following station-function technique wherein two complete functions are employed instead of the usual one function. The first function is used from 0°–17° angle of attack, and the second function is used from 17° to about 100° angle of attack.

First function

By letting $x = \alpha/20$ and utilizing the functions from Table 10, the resulting final equation becomes

$$c_n = \left[2.5825\left(\frac{\alpha}{20}\right) - 4.6240\left(\frac{\alpha}{20}\right)^2 + 13.4740\left(\frac{\alpha}{20}\right)^3 - 12.4360\left(\frac{\alpha}{20}\right)^4\right] \quad (17)$$

Second function

By letting $f(x) = f(\alpha) - 0.55$ and $x = [(\alpha - 15)/20]$, and by utilizing the functions of Eq. (13), the resulting equation becomes

$$c_n = \left[0.550000 + 0.785206\left(\frac{\alpha - 15}{20}\right) + 0.114957\left(\frac{\alpha - 15}{20}\right)^2 - 0.160206\left(\frac{\alpha - 15}{20}\right)^3 + 0.0389810\left(\frac{\alpha - 15}{20}\right)^4 - 0.00312650\left(\frac{\alpha - 15}{20}\right)^5\right] \quad (18)$$

Table 1 Two functions

$x = 0.1, 0.2$
$f_1(x) = 20.0x - 100.0x^2$
$f_2(x) = -5.00x + 50.0x^2$

Table 2 Two functions

$x = 0.2, 0.4$
$f_1(x) = 10.0x - 25.0x^2$
$f_2(x) = -2.50x + 12.5x^2$

Table 3 Two functions

$x = 0.4, 0.9$
$f_1(x) = 4.500x - 5.000x^2$
$f_2(x) = -0.8890x + 2.220x^2$

Table 4 Two functions

$x = 0.8, 1.4$
$f_1(x) = 2.9167x - 2.0833x^2$
$f_2(x) = -0.95238x + 1.1905x^2$

Table 5 Three functions

$x = 0.1, 0.2, 0.3$
$f_1(x) = 30.00x - 250.0x^2 + 500.0x^3$
$f_2(x) = -15.00x + 200.0x^2 - 500.0x^3$
$f_3(x) = 3.333x - 50.00x^2 + 166.7x^3$

Table 6 Three functions

$x = 0.2, 0.5, 0.9$
$f_1(x) = 10.714x - 33.333x^2 + 23.810x^3$
$f_2(x) = -3.0000x + 18.333x^2 - 16.666x^3$
$f_3(x) = 0.39680x - 2.7770x^2 + 3.9680x^3$

Table 7 Three functions

$x = 0.30, 0.65, 1.0$
$f_1(x) = 8.846x - 22.45x^2 + 13.61x^3$
$f_2(x) = -3.768x + 16.33x^2 - 12.56x^3$
$f_3(x) = 0.7960x - 3.877x^2 + 4.082x^3$

Table 8 Three functions

$x = 0.4, 0.7, 1.0$
$f_1(x) = 9.722x - 23.61x^2 + 13.89x^3$
$f_2(x) = -6.349x + 22.22x^2 - 15.89x^3$
$f_3(x) = 1.556x - 6.111x^2 + 5.556x^3$

Table 9 Four functions

$x = 0.2, 0.4, 0.7, 1.0$
$f_1(x) = 17.49x - 86.21x^2 + 131.2x^3 - 62.48x^4$
$f_2(x) = -9.720x + 72.19x^2 - 131.9x^3 + 69.42x^4$
$f_3(x) = 2.540x - 21.58x^2 + 50.77x^3 - 31.73x^4$
$f_4(x) = -0.3900x + 3.470x^2 - 9.020x^3 + 6.940x^4$

Table 10 Four functions

$x = 0.20, 0.45, 0.70, 1.0$
$f_1(x) = 15.7440x - 73.2430x^2 + 107.488x^3 - 49.9950x^4$
$f_2(x) = -9.05000x + 67.2260x^2 - 122.816x^3 + 64.6400x^4$
$f_3(x) = 3.42800x - 28.1880x^2 + 62.8510x^3 - 38.0920x^4$
$f_4(x) = -0.477000x + 4.12800x^2 - 10.2260x^3 + 7.57500x^4$

Table 11 Five functions

$x = 1, 2, 3, 4, 5$

$$f_1(x) = 5.000000x - 6.417000x^2 + 2.958333x^3 - 0.5833333x^4 + 0.04166667x^5$$
$$f_2(x) = -5.000000x + 8.916667x^2 - 4.916667x^3 + 1.083333x^4 - 0.08333333x^5$$
$$f_3(x) = 3.333333x - 6.500000x^2 + 4.083333x^3 - 1.000000x^4 + 0.08333333x^5$$
$$f_4(x) = -1.245000x + 2.541667x^2 - 1.708333x^3 + 0.4583333x^4 - 0.04166667x^5$$
$$f_5(x) = 0.2000000x - 0.4166667x^2 + 0.2916667x^3 - 0.08333333x^4 + 0.008333333x^5$$

Table 12 Ten functions

$x = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$

$$f_1(x) = 10.001928x - 19.295084x^2 + 15.861844x^3 - 7.3222693x^4 + 2.0989164x^5 - 0.38856085x^6 + 0.046572496x^7 - 0.0034927316x^8 + 0.00014900696x^9 - 0.0000027599220x^{10}$$
$$f_2(x) = -22.508548x + 54.67541x^2 - 51.778348x^3 + 26.282956x^4 - 8.0590114x^5 + 1.5679481x^6 - 0.19516359x^7 + 0.015072191x^8 - 0.00065811765x^9 + 0.000012419375x^{10}$$
$$f_3(x) = 40.022460x - 103.88834x^2 + 106.04424x^3 - 57.415944x^4 + 18.555972x^5 - 3.7667894x^6 + 0.48524326x^7 - 0.038538378x^8 + 0.0017218772x^9 - 0.000033117604x^{10}$$
$$f_4(x) = -52.538717x + 140.75433x^2 - 149.47441x^3 + 84.401428x^4 - 28.380005x^5 + 5.9685234x^6 - 0.79291207x^7 + 0.064663171x^8 - 0.0029553546x^9 + 0.000057954529x^{10}$$
$$f_5(x) = 50.445755x - 137.66822x^2 + 149.76957x^3 - 36.961333x^4 + 30.114610x^5 - 6.5193375x^6 + 0.88995034x^7 - 0.074399906x^8 + 0.0034769088x^9 - 0.000069543902x^{10}$$
$$f_6(x) = -35.037542x + 96.785759x^2 - 107.01422x^3 + 63.366188x^4 - 22.433697x^5 + 4.9718075x^6 - 0.69496813x^7 + 0.059452414x^8 - 0.0028394934x^9 + 0.000057951974x^{10}$$
$$f_7(x) = 17.163974x - 47.821080x^2 + 53.492910x^3 - 32.135591x^4 + 11.571700x^5 - 2.6137966x^6 + 0.37291051x^7 - 0.032583226x^8 + 0.0015894645x^9 - 0.000033114684x^{10}$$
$$f_8(x) = -5.6327929x + 15.794159x^2 - 17.822390x^3 + 10.825779x^4 - 3.9506709x^5 + 0.90636883x^6 - 0.13160439x^7 + 0.011722418x^8 - 0.00058363548x^9 + 0.000012417732x^{10}$$
$$f_9(x) = 1.1128149x - 3.1357269x^2 + 3.5625481x^3 - 2.1829512x^4 + 0.80524141x^5 - 0.18713612x^6 + 0.027585820x^7 - 0.0025002063x^8 + 0.00012693818x^9 - 0.0000027594353x^{10}$$
$$f_{10}(x) = -0.10016758x + 0.28336643x^2 - 0.32369416x^3 + 0.19974809x^4 - 0.074335843x^5 + 0.017463164x^6 - 0.0026080541x^7 + 0.00024009423x^8 - 0.000012417958x^9 + 0.0000027593744x^{10}$$

The use of two polynomials in defining such a curve as Fig. 6 is suitable for the automatic digital computer, since the computed angle of attack can be used as the selective intelligence defining the polynomial to be used.

The foregoing station-function polynomials not only appreciably reduced the computer time but provided the desired accuracy in the blade-motion solutions resulting in values comparable with measured experimental data.

Polynomial Representation of Helicopter Longitudinal Control

The calculations in determining the characteristics for trim at various air speeds in steady and accelerated flight conditions require a statement of the relation between control surface deflection and cockpit control movement. For linear relations this statement is simply applied. For a nonlinear relation, such as that shown by Fig. 7, it is convenient to represent blade angle with control stick position by an algebraic polynomial. This can be simply determined through use of a set of functions provided by the station-function technique. The functions employed are from Table 10. By letting $x = (\theta_2' + 6)/12$ and $y = (\theta_2 + 3.5°)$, the final equation becomes

$$\theta_2 = [-1.1485 + 0.88380\theta_2' + 0.075910(\theta_2')^2 - 0.00095500(\theta_2')^3 + 0.0000095970(\theta_2')^4] \quad (19)$$

The solid curve of Fig. 7 is the control motion to be represented, and the circled points are those calculated by Eq. (19). Excellent agreement, of course, results from use of such a high-degree equation for this simple curve.

Conclusions

The examples given in the previous section "Engineering Applications" numerically illustrate the method and flexibility of the station-function technique in polynomial description of several functions often used in the solution of many engineering problems. The examples shown were selected to illustrate the handling of diverse arrangements of boundary values and function singularities.

The simplicity and diverse applications of the station functions for obtaining reasonably accurate descriptive interpolation functions is the purpose of this presentation. It is hoped that the tables of functions included in this paper may prove as helpful to others as they have to the author and his colleagues.

Tables of Functions

The functions shown in Tables 1–11 have been in use extensively. The tenth-degree functions, Table 12, have been included to extend the usefulness of this technique for polynomial description of the higher-order functions encountered in some engineering problems.

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